**Chapter 8**

**Interval Estimation**

**Learning Objectives**

1. Know how to construct and interpret an interval estimate of a population mean and / or a population proportion.

2. Understand and be able to compute the margin of error.

3. Learn about the *t* distribution and its use in constructing an interval estimate whenis unknown for a population mean.

4. Be able to determine the size of a simple random sample necessary to estimate a population mean and/or a population proportion with a specified margin of error.

5. Know the definition of the following terms:

confidence interval margin of error

confidence coefficient degrees of freedom

confidence level

**Solutions:**

1. a. 

b. At 95%, 

2. a. 32  1.645 

32  1.4 or 30.6 to 33.4

b. 32  1.96 

32  1.66 or 30.34 to 33.66

c. 32  2.576 

32  2.19 or 29.81 to 34.19

3. a. 80  1.96 

80  3.8 or 76.2 to 83.8

b. 80  1.96 

80  2.68 or 77.32 to 82.68

c. Larger sample provides a smaller margin of error.

1. Sample mean 

Margin of Error = 160 – 156 = 4





*n* = (7.35)2 = 54

5. a. With 99% confidence 

Margin of Error = 

b. Confidence Interval: 21.52  1.93 or 19.59 to 23.45

6. 

8.5  1.96(3.5/)

8.5  .4 or 8.1 to 8.9

7. Margin of error = 

  

Margin of error = 1.96(600/) = 166.31

A larger sample size would be needed to reduce the margin of error to $150 or less. Section 8.3 can be used to show that the sample size would need to be increased to *n* = 62.



Solving for *n* shows *n* = 62

8. a. Since *n* is small, an assumption that the population is at least approximately normal is required so that the sampling distribution of can be approximated by a normal distribution.

b. Margin of error: 

c. Margin of error: 

9.   

33.5  1.96

33.5  2.8 or 30.7 to 36.3 hours

10. a. 

119,155  1.645

119,155  5517 or $113,638 to $124,672

b. 119,155  1.96 

119,155  6574 or $112,581 to $125,729

c. 119,155  2.576 

119,155  8640 or $110,515 to $127,795

1. The confidence interval gets wider as we increase our confidence level. We need a wider interval to be more confident that it will contain the population mean.

11. a. .025

b. 1 - .10 = .90

c. .05

d. .01

e. 1 – 2(.025) = .95

f. 1 – 2(.05) = .90

12. a. 2.179

b. -1.676

c. 2.457

d. Use .05 column, -1.708 and 1.708

e. Use .025 column, -2.014 and 2.014

13. a. 

b.

|  |  |  |
| --- | --- | --- |
|  |  |  |
| 10 | 0 | 0 |
| 8 | -2 | 4 |
| 12 | 2 | 4 |
| 15 | 5 | 25 |
| 13 | 3 | 9 |
| 11 | 1 | 1 |
| 6 | -4 | 16 |
| 5 | -5 | 25 |
|  |  | 84 |



c. 

d. 

10 ± 2.9 or 7.1 to 12.9

14.  *df* = 53

a. 22.5 ± 1.674

22.5 ± 1 or 21.5 to 23.5

b. 22.5 ± 2.006

22.5 ± 1.2 or 21.3 to 23.7

c. 22.5 ± 2.672

22.5 ± 1.6 or 20.9 to 24.1

d. As the confidence level increases, there is a larger margin of error and a wider confidence interval.

15. 

90% confidence *df* = 64 *t*.05 = 1.669

19.5 ± 1.669

19.5 ± 1.08 or 18.42 to 20.58

95% confidence *df* = 64 *t*.025 = 1.998

19.5 ± 1.998

19.5 ± 1.29 or 18.21 to 20.79

16. a. Using Minitab or Excel, = 9.7063 and *s* = 7.9805

The sample mean years to maturity is 9.7063 years with a standard deviation of 7.9805.

b.  *df* = 39 *t*.025 = 2.023

9.7063 ± 2.023

9.7063 ± 2.5527 or 7.1536 to 12.2590

The 95% confidence interval for the population mean years to maturity is 7.1536 to 12.2590 years.

c. Using Minitab or Excel, = 3.8854 and *s* = 1.6194

The sample mean yield on corporate bonds is 3.8854% with a standard deviation of 1.6194.

d.  *df* = 39 *t*.025 = 2.023

3.8854 ± 2.023

3.8854 ± .5180 or 3.3674 to 4.4034

The 95% confidence interval for the population mean yield is 3.3674 to 4.4034 percent.

17. Using Minitab or Excel, = 6.34 and *s* = 2.163

 *df* = 49 *t*.025 = 2.010

6.34 ± 2.010

6.34 ± .61 or 5.73 to 6.95

18. For the JobSearch data set,

and *s* = 11.8862

a. = 22 weeks

b. margin of error = 3.8020

c. The 95% confidence interval is margin of error

22  3.8020 or 18.20 to 25.80

d. Skewness = 1.0062, data are skewed to the right. Use a larger sample next time.

19. a.  *df* = 44

*t*.025 = 2.015 *s* = 65

2.015= 19.52 or approximately $20

b. 

273 ± 20 or 253 to 293

c. At 95% confidence, the population mean is between $253 and $293. This is definitely above the $229 level of 2 years ago. Hotel room rates are increasing.

The point estimate of the increase is $273 - $229 = $44 or 19%.

20. 



  *t*.025 *df* = 19

22.00  2.093 

22.00  .52 or 21.48 to 22.52 minutes

21. liters of alcoholic beverages



*t*.025 = 2.093 *df* = 19

95% confidence interval:   *t*.025

130  2.093

130  30.60 or 99.40 to 160.60 liters per year

22. a. 

The point estimate of the population mean ticket sales revenue per theater is $10,905.



95% confidence interval:   *t*.025with *df* = 24 

10,905 + 

10,905 + 1636

The 95% confidence interval for the population mean is $9,269 to $12,541. We are 95% confident that the population mean three-day tickets sales revenue per theater is between $9,269 and $12,541.

b. Mean number of customers per theater = 10,905/7.16 = 1523

c. Total number of customers = 3118(1523) = 4,748,714

Total box office ticket sales for the three-day weekend = 3118(10,905)  $34 million

23. 

24. a. Planning value of ** = Range/4 = 36/4 = 9

b. 

c. 

25. 



26. a.  Use 25.

If the normality assumption for the population appears questionable, this should be adjusted upward to at least 30.

b.  Use 49 to guarantee a margin of error no greater than .07. However, the US EIA may choose to increase the sample size to a round number of 50

c.  Use 97

For reporting purposes, the US EIA might decide to round up to a sample size of 100.

27. Planning value 

a. 

b. 

c. 

d. Sampling 5403 college graduates to obtain the $100 margin of error would be viewed as too expensive and too much effort by most researchers.

28. a. 

b. 

c. 

d. The sample size gets larger as the confidence is increased. We would not recommend 99% confidence. The sample size must be increased by 137 = 465 - 328 to go from 90% to 95%. This may be reasonable. However, increasing the sample size by 338 = 803 - 465 to go from 95% to 99% would probably be viewed as too expensive and time consuming for the 4% gain in confidence.

29. a. 

b. 

30. Planning value from previous study: 



Use *n* = 1537 to guarantee the margin of error will not exceed 100.

31. a.  = 100/400 = .25

b. 

c. 

.25  1.96 (.0217)

.25  .0424 or .2076 to .2924

32. a. .70  1.645 

.70  .0267 or .6733 to .7267

b. .70  1.96

.70  .0318 or .6682 to .7318

33. 

34. Use planning value *p*\* = .50



35. a.  = 1760/2000 = .88

b. Margin of Error



c. Confidence interval:



or .868 to .892

d. Margin of Error



95% Confidence Interval

.88 + .0142 or .8658 to .8942

36. a.  = 46/200 = .23

b. 



.23  1.96(.0298)

.23  .0584 or .1716 to .2884

37. a.  = 473/1100 = .43

b. 

c.  ± .0293

.43  .0293 or .4007 to .4593

d. With roughly 40% to 46% of employees surveyed indicating strong dissatisfaction and with the high cost of finding successors, employers should take steps to improve employee satisfaction. The survey suggested employers may anticipate high employee turnover costs if employee dissatisfaction remains at the current level.

38. a. 

b. = .6420

Margin of error = 

Confidence interval: .6420  .0738 or .5682 to .7158

c.  Use *n* = 354

39. a. 

b.  Use 971

40. Margin of error: 

95% Confidence interval:  ± .0346

.52  .0346 or .4854 to .5546

41. a. Margin of error = 

Interval estimate: .87 ± .0190 or .851 to .889

b. Margin of error = 

Interval estimate: .75 ± .0245 or .7255 to .7745

c. The margin of error is larger in part (b). This is because the sample proportion is closer to .50 in part (b) than in part (a). This also leads to a larger interval estimate in part (b).

42. a. 

= 1.96(.0226) = .0442

b. 

September  Use 601

October  Use 1068

November 

Pre-Election 

43. a. Margin of Error =

95% Confidence Interval: .53  .0253 or .5047 to .5553

b. Margin of Error = 1.96= .0234

95% Confidence Interval: .31  .0234 or .2866 to .3334

c. Margin of Error = 1.96= .0110

95% Confidence Interval: .05  .0110 or .039 to .061

d. The margin of error decreases asgets smaller. If the margin of error for all of the interval estimates must be less than a given value (say .03), an estimate of the largest proportion should be used as a planning value. Usingas a planning value guarantees that the margin of error for all the interval estimates will be small enough.

44. a. Margin of error: 

b. Confidence interval:  margin of error

33.77  4.00 or $29.77 to $37.77

45. a.  *t*.025 *df* = 63 *t*.025 = 1.998

252.45  1.998

252.45  18.61 or $233.84 to $271.06

b. Yes. the lower limit for the population mean at Niagara Falls is $233.84 which is greater than $215.60.

46. a. Margin of error = *t*.025

*df* = 79 *t*.025 = 1.990 *s* = 550

1.990= 122

b. ± margin of error

1873  or $1751 to $1995

c. As of March, 2008, 92 million Americans were of age 50 and over

Estimate of total expenditures = 92(1873) = 172,316

In dollars, we estimate that $172,316 million dollars are spent annually by Americans of age 50 and over on restaurants and carryout food.

d. We would expect the median to be less than the mean of $1873. The few individuals that spend much more than the average cause the mean to be larger than the median. This is typical for data of this type.

47. a. From the sample of 30 stocks, we find= 21.9 and *s* = 14.86

A point estimate of the mean P/E ratio for NYSE stocks on January 19, 2004 is 21.9.

Margin of error =

95% Confidence Interval: 21.9  5.5 or 16.4 to 27.4

b. The point estimate is greater than 20 but the 95% confidence interval goes down to 16.4. So we would be hesitant to conclude that the population mean P/E ratio was greater than 20. Perhaps taking a larger sample would be in order.

c. From the sample of 30 stocks, we find= 21/30 = .70

A point estimate of the proportion of NYSE stocks paying dividends is .70.

With = 30(.7) = 21 and = 30(.3) = 9, we would be justified in using a normal distribution to construct a confidence interval. A 95% confidence interval is





.7 .16 or .54 to .86

While the sample size is large enough to use the normal distribution approximation, the sample size is not large enough to provide much precision. The margin of error is larger than most people would like.

48.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Variable | N | Mean | StDev | SE Mean | 95.0% CI |
| Time | 150 | 14.000 | 3.838 | 0.313 | (13.381, 14.619) |

a. = 14 minutes

b. 13.381 to 14.619

c. 7.5 hours = 7.5(60) = 450 minutes per day

An average of 450/14 = 32 reservations per day if no idle time. Assuming perhaps 15% idle time or time on something other than reservations, this could be reduced to 27 reservations per day.

d. For large airlines, there are many telephone calls such as these per day. Using the online reservations would reduce the telephone reservation staff and payroll. Adding in a reduction in total benefit costs, a change to online reservations could provide a sizeable cost reduction for the airline.

49. a. Using Excel or Minitab: 

The amount of playing time is approximately 11 minutes. So the time standing around is over 6 times as much. You may find this difference surprising; the authors did.

b. Using Excel or Minitab: *s* = 4.4943 minutes

c.  *df* = 59 = 2.001

66.93  2.001 (4.4943/)

66.93  1.16 or 65.77 to 68.09

50. 

51. 



52. 

53. a. 

.47  1.96 

.47  .0461 or .4239 to .5161

b. .47  2.576

.47  .0606 or .4094 to .5306

c. The margin of error becomes larger.

54. a.  = 200/369 = .5420

b. 

c. .5420  .0508 or .4912 to .5928

55. a.  = .74

Margin of error =

95% Confidence Interval: .74  .02 or .72 to .76

b.  = .48

Margin of error =

95% Confidence Interval: .48  .03 or .45 to .51

c. The margin of error is larger in part b for two reasons. With  = .48, the estimate of the standard error is larger. And= .2576 is larger than = 1.96

56. a. = 455/550 = .8273

b. Margin of error

95% Confidence interval: .8273  .0316 or .7957 to .8589

57. a. 

b.  = 520/2017 = .2578

c. 

.2578  1.96 

.2578  .0191 or .2387 to .2769

58. a. 

b. 

59. a. Government industry 95% margin of error: 

95% Confidence interval: .37.0669 or .3031 to .4369

Health care industry 95% margin of error: 

95% Confidence interval: .33.0652 or .2648 to .3952

Education industry 95% margin of error: 

95% Confidence interval: .28.0622 or .2178 to .3422

b. The government industry has the largest margin of error. So the sample size must be large enough to reduce the margin of error for the government industry to .05 or less. Using the proportions found in the study reported in *USA Today* as a planning value



This is an increase of 159 workers surveyed in each industry. The added cost of the larger sample size would have to be taken into account before deciding whether the smaller margin of error is worth the added cost of obtaining the data.

60. a.  = 618/1993 = .3101

b. 

.3101  1.96 

.3101  .0203 or .2898 to .3304

c. 



No; the sample appears unnecessarily large. The .02 margin of error reported in part (b) should provide adequate precision.